Fusing Acoustic Ranges and Inertial Measurements in AUV Navigation: the Typhoon AUV at CommsNet13 Sea Trial

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***Abstract*—The paper presents some experimental results of autonomous underwater navigation, based on the fusion of acoustic and inertial measurements. The work is in the framework of the Thesaurus project, funded by the Tuscany Region, aiming at developing techniques for systematic exploration of marine areas of archaeological interest through a team of Autonomous Underwater Vehicles (AUVs). The test was carried out with one Typhoon vehicle, a 300m depth rated AUV with acoustic communication capabilities, during the CommsNet13 experiment, organized and scientifically coordinated by the NATO S&T Org. Ctr. for Maritime Research and Experimentation (CMRE, formerly NURC), with the participation of several research institutions. The fusion algorithm is formally casted into an optimal stochastic filtering problem, where the rough estimation of the vehicle position, velocity and attitude, are refined by using the depth measurement, the relative measurements available on the acoustic channel and the vehicle surge speed.**

I. Introduction

The problem of navigation and self-localization is particullary challenging in an underwater context due to the tight environmental constraints, such as the absence of an absolute positioning system, i.e. GPS, and the small communication bandwidth of the acoustic channel.

A thorough review on Autonomous Underwater Vehicle (AUV) navigation can be found in [1] and references within. As there stated, many techniques have been investigated and classified in three main categories, depending on the purpose of the sensors employed: inertial navigation, acoustic navigation and geophisical navigation [2]–[4]. To support navigation, AUV commonly mount on-board sensors of different nature, both inertial and acoustic. Sensor data fusion plays so a fundamental role in achieving a navigation accuracy such that a vehicle can perform an underwater mission autonomously. Basically, the Inertial Measurement Unit (IMU) provides information that allow to continuously calculate the vehicle position, velocity and orientation via dead reckoning. Since dead reckoning is subject to cumulative errors due to the IMU inaccuracies, the navigation status of the vehicle needs to be periodically fixed with measurements from other more precise sensors, e.g. Doppler Velocity Log (DVL), depth sensor, compass, Ultra-Short Base Line (USBL).

Most of data integration methods underlie linear or nonlinear state estimators. Many examples of position and velocity estimation via sensor fusion algorithm can be found in literature. Particularly interesting are the experimental results reported in some of these works. In [5] and [6] the measurement integration process is realized using a particle filter. The first combines DVL data with USBL observations to obtain the trajectory estimate of a lawn-mower path mission, showing how USBL corrections are able to reduce the long-term drift of the DVL estimate. The simultaneous equipment of a DVL and an USBL can however be too much expensive on a low-cost vehicle. In [6] the authors improve dead-reckoning navigation by integrating USBL measurements, proving even in this case the benefit of sensor fusion on experimental data. The results obtained in the previous works are relevant in terms of the estimate error, but the computational cost required by a particle filter is considerable. For instance, the approach presented in [6] relies in a dedicated CPU for the filtering algorihtm. In [7] an Extended Kalman Filter (EKF) is implemented to tighly couple the inertial-based estimate with USBL differential range and time measurements. Performance is reported on experimental data sets with a large number of USBL fixes.

In our previous work [8], we report a preliminary analysis of some raw navigation data, addressing the potential of using them in data fusion procedures to improve localization accuracy and navigation capabilities. The data were collected with one Typhoon vehicle, shown in Figure 1, a $300m$ depth rated low-cost AUV with acoustic communication capabilities [9], during the CommsNet13 experiment, which took place in September 2013 in the La Spezia Gulf, North Tyrrhenian Sea. The experiment was organized and scientifically coordinated by the NATO S&T Org. Ctr. for Maritime Research and Experimentation (CMRE, formerly NURC), with the participation of several research institutions; it included among its objectives the evaluation of on-board acoustic USBL systems for navigation and localization of AUVs. In the presented experiment, the vehicle was equipped with a USBL modem and a low-cost, low-accuracy IMU.

Fig. 1. Typhoon AUV during CommsNet13 experiment

The USBL modem could communicate with a fixed sensors network consisting of four acoustic modems, placed on the seabed and cable-connected to the shore so that they could be continuously operated and monitored. The vehicle could use its USBL modem to estimate its relative position with respect to the fixed installation. The role of the Typhoon in this experimentation was to perform both surface and underwater navigation in autonomous modality, while trying to localize itself with respect to the known positions of the fixed modems, by employing the USBL measurements. It is worth to notice that the acoustic positioning observations can not be received very frequently or at a constant rate when the communication is performed within a sensor network, due to the overhead of the network itself. This work takes the previous elaboration one step further, presenting some experimental results on underwater navigation, based on the fusion of the acoustic and inertial measurements. The fusion algorithm is formally casted into an optimal stochastic filtering problem, where the rough estimation of the vehicle position and velocity are refined by using the relative measurements available on the acoustic channel and the surge speed estimated as a function of the longitudinal propeller thrust.

The paper is organized as follows: in Section II the model of the system is developed and Section III gives details about the proposed filtering method. Section IV shows the main results obtained on the data collected on Sept. 12. Finally, Section V reports the conclusions.

II. Modeling Aided-Inertial Navigation

The inertial mechanization equations [10] are a set of non-linear differential equations relating vehicles attitude, velocity and position – the *state* of the system – to known/measured inertial quantities. In the general theory of strapdown inertial navigation, the equations are integrated given the measurements of inertial sensors, accelerometers and gyroscopes, measured in the body frame, which usually represent the inputs of the navigation system. For the purposes of this work, the classical formulation of the inertial mechanization equation is simplified, due to some assumptions made. The main one is that the navigation frame (NED) can be considered not to change its orientation with respect to the global (ECEF) frame, during the whole navigation task. This means that the Earth is approximated as a flat surface in the neighborhood of the starting point. By putting the NED reference frame on the Earth surface at the location corresponding to the starting point, the set of the inertial navigation equations can be transformed in local coordinates. This is a realistic assumption in the framework of the proposed work, since the motion of the vehicle was assumed to be enclosed inside a *small enough* area, such that a sufficient number of acoustic measurements could be detected. These were the conditions actually met during the experimental tests conducted. Moreover, the attitude of the vehicle was assumed known. More specifically, the attitude $\boldsymbol{\theta}$ was *pseudo-measured* by the inertial unit on-board the vehicle, via integration of the gyroscopes measurements together with the sensed gravity and the Earth magnetic field measurements in an Attitude-Heading Reference System (AHRS) fashion [11], [12].

Upon the above hypothesis, the *local* continuous-time navigation equations in the NED frame can be written as:

$$\begin{aligned} \dot{\mathbf{p}}_n &= \mathbf{v}_n \\ \dot{\mathbf{v}}_n &= {}^n\mathbf{R}_b(\boldsymbol{\theta})\left(\mathbf{a}_b - \boldsymbol{\epsilon}_b + \boldsymbol{\nu}_{\boldsymbol{a}}\right) + \mathbf{g}_n \\ \dot{\boldsymbol{\epsilon}}_b &= \boldsymbol{\nu}_{\boldsymbol{\epsilon}} \end{aligned} \tag{1}$$

where $\mathbf{p}_n$ and $\mathbf{v_n}$ are respectively the position and velocity of the system, both expressed in the NED frame, whereas ϵ_b denotes the the accelerometers bias term. Since we assume not to have a prior information regarding the nature of bias time evolution, their dynamics were modeled as random walks, where $\boldsymbol{\nu}_{\boldsymbol{\epsilon}} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_{\boldsymbol{\epsilon}})$ is a zero-mean white noise with constant variance. Note that the velocity dynamics employ the body accelerations measured by the accelerometers depurated by the bias term and then converted in the local navigation frame through the transformation matrix ${}^n\mathbf{R}_b(\boldsymbol{\theta})$. Finally, the vehicle acceleration is obtained by compensating for the gravity term $\mathbf{g}$. The body-to-navigation matrix ${}^n\mathbf{R}_b$ is evaluated based on the vehicle attitude $\boldsymbol{\theta}$, expressed in Euler angles, computed by the inertial sensors suite. The random variable $\boldsymbol{\nu}_{\boldsymbol{a}} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_{\mathbf{a}})$ accounts for the noise that intrinsically affects the inertial sensor. This latter noise term was supposed white as well.

In this work it is assumed that the navigation system relies on the measurements from the USBL, depth sensor and surge velocity, pseudo-measured from the propellers thrust [8]. The further measurement from the GPS device was used as a navigation aid in the first time instants only, to initialize the navigation algorithm. Then, this latter measurements were used as ground truth only, meaning that the navigation system did not take advantage of the global localization system during the normal operation phase. The measurement equations can be thus written as:

$$\boldsymbol{y}_{\text{gps}} = \mathbf{p}_{\text{gps},n} + \boldsymbol{\eta}_{\text{gps}} \tag{2}$$

$$\mathbf{y}_{\text{usbl}} = \mathbf{p}_{\text{m}_i,n} - {}^n\mathbf{R}_b(\boldsymbol{\theta})\,{}^b\mathbf{R}_u\left(\mathbf{p}_{\text{u-m}_i,u} + \boldsymbol{\eta}_{\text{usbl}}\right) \tag{3}$$

$$y_{\text{depth}} = p_{\text{depth},n} + \eta_{\text{depth}} \tag{4}$$

$$\mathbf{y}_{\text{prop}} = {}^n\mathbf{R}_b(\boldsymbol{\theta})\left(\mathbf{v}_{\text{prop},b} + \boldsymbol{\eta}_{\text{prop}}\right) \tag{5}$$

Each measurement is affected by a measurement noise, denoted with the η symbol, that we assume white, with zero mean and constant covariance matrix $\mathbf{R}_{\text{gps}}$, $\mathbf{R}_{\text{usbl}}$, R_{depth} and $\mathbf{R}_{\text{prop}}$ respectively. The GPS measure contains the N-E coordinates of the vehicle position in the local navigation frame with respect to the starting position, computed employing the flat

Earth surface approximation [10]. In (3), the actual measure from the USBL is the relative position of the vehicle with respect to the i-th acoustic source, namely $\mathbf{p}_{\text{u-m}_i,u}$, expressed in the coordinate frame of the device. In order to make such measurement reflect the absolute position of the vehicle, we first convert the measure in the body-fixed frame through the transformation matrix ${}^b\mathbf{R}_u$ and then in the local navigation frame with the matrix ${}^n\mathbf{R}_b(\boldsymbol{\theta})$. Finally, we compose the result with the absolute position of the i-th acoustic source $\mathbf{p}_{\text{m}_i,n}$ to obtain the absolute vehicle position. The depth sensor evaluates the vertical direction (D) of the absolute vehicle position. Finally, the surge speed of the vehicle $v_{\text{prop},b}$ is roughly estimated from the propellers thrust, which is supposed aligned with the vehicle nose, as in [8]. Thus, we obtain the vehicle velocity in NED coordinates by transforming the surge speed through the matrix ${}^n\mathbf{R}_b(\boldsymbol{\theta})$, eq. (5). Note that $\mathbf{v}_{\text{prop},b} = \begin{bmatrix} v_{\text{prop},b} & 0 & 0 \end{bmatrix}^T$. In order to fuse such measurements with the inertial navigation system, it is convenient to make explicit the dependence of the above measurements with the inertial mechanization states. For this reason, we can write:

$$\boldsymbol{y}_{\text{gps}} = \begin{bmatrix} \mathbf{I}_2 & \mathbf{0}_{2\times7} \end{bmatrix} \boldsymbol{x} + \boldsymbol{\eta}_{\text{gps}} \tag{6}$$

$$\boldsymbol{y}_{\text{usbl}} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{0}_{3\times6} \end{bmatrix} \boldsymbol{x} + {}^n\mathbf{R}_b(\boldsymbol{\theta})\,{}^b\mathbf{R}_u \boldsymbol{\eta}_{\text{usbl}} \tag{7}$$

$$y_{\text{depth}} = \begin{bmatrix} 0 & 0 & 1 & \mathbf{0}_{1\times6} \end{bmatrix} \boldsymbol{x} + \eta_{\text{depth}} \tag{8}$$

$$\boldsymbol{y}_{\text{prop}} = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{I}_3 & \mathbf{0}_{3\times3} \end{bmatrix} \boldsymbol{x} + {}^n\mathbf{R}_b(\boldsymbol{\theta}) \boldsymbol{\eta}_{\text{prop}} \tag{9}$$

For notational simplicity, the model (1) together with the above measurement model can be written in a more compact form, that is:

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}) + g(\mathbf{x}, \mathbf{u})\, \boldsymbol{\nu} \tag{10}$$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + J(\boldsymbol{\theta})\, \boldsymbol{\eta} \tag{11}$$

III. Filter Design

According to the motion and sensitivity parameters dynamics in Equation (1), given the defined measurements model, an Extended Kalman Filter [13] was designed and tested. The aim of the filter is to simultaneously estimate both navigation variables, position and velocity of the vehicle, and the accelerometers bias term. The filtering process is composed by two steps: in the *prediction* step, a rough estimation of the state is obtained by evolving the dynamical model (1) of the system, by using the measured inertial measurements. The second step is the *correction* step, which is executed when a new measurement is available from one or more of the auxiliary sensors, in order to reduce the error of the predicted state estimate.

For the purpose of numerical implementation, the motion and measurements model equations were time-discretized using the Euler integration method. The model of the system can be thus rewritten in the following compact form:

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k) + g(\boldsymbol{x}_k, \mathbf{u}_k)\, \boldsymbol{\nu}_k \tag{12}$$

$$\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + J(\boldsymbol{\theta}_k)\, \boldsymbol{\eta}_k \tag{13}$$

Here, the subscript k indicates that the quantity is referred to the k-th time instant.

The prediction step is executed at each time instant $k > 0$ when a new inertial measurement is made available, by calculating the predicted state $\hat{\boldsymbol{x}}^-_{k+1}$ and the predicted covariance matrix of the estimation error $\mathbf{P}^-_{k+1}$, starting from the initial conditions $\hat{x}_0$, $\mathbf{P}_0$:

$$\begin{aligned} \hat{\boldsymbol{x}}^-_{k+1} &= f\left(\hat{\boldsymbol{x}}^+_k, \mathbf{u}_k\right) \\ \mathbf{P}^-_{k+1} &= \mathbf{F}_k \mathbf{P}^+_k \mathbf{F}^T_k + \mathbf{G}_k \mathbf{Q} \mathbf{G}^T_k \end{aligned} \tag{14}$$

The variables with the '+' superscript in the prediction equation are the refined estimations of the state and of the covariance matrix at the previous time step, obtained by running a Kalman update step with the past measurements from the available auxiliary sensors. In (14), the matrices $\mathbf{F}_k$ and $\mathbf{G}_k$ are obtained from (10) as follows:

$$\mathbf{F}_k = \left.\frac{\partial f(\boldsymbol{x}, \mathbf{u})}{\partial \boldsymbol{\xi}}\right|_{\hat{\boldsymbol{x}}^+_k, \mathbf{u}_k}, \quad \mathbf{G}_k = g\left(\hat{\boldsymbol{x}}^+_k, \mathbf{u}_k\right)$$

and $\mathbf{Q} = diag\left(\mathbf{Q}_{\mathbf{a}}, \mathbf{Q}_{\epsilon}\right)$ is the noise covariance matrix.

When a new measurement from the auxiliary sensors is available, the Kalman update step is executed, in order to correct the prediction obtained in the current step.

It is worth to mention that each measurement has its own notification rate, thus it usually happens that at a given time step, not all the measurements can be available. Under these assumptions, the correction step is made by employing the available measurements at the current time and by selecting the entries in the matrices of the output model which correspond with the currently available maesurements. The correction step is then executed by calculating the state estimate $\hat{\boldsymbol{x}}^+_{k+1}$ and the covariance matrix of the estimate error $\mathbf{P}^+_{k+1}$ as:

$$\begin{aligned} \hat{\boldsymbol{x}}^+_{k+1} &= \hat{\boldsymbol{x}}^-_{k+1} + \mathbf{K}_k \left(\boldsymbol{y}_k - \mathbf{H}_k \hat{\boldsymbol{x}}^-_{k+1}\right) \\ \mathbf{P}^+_{k+1} &= \left(\mathbf{I} - \mathbf{K}_k \mathbf{H}_k\right) \mathbf{P}^-_{k+1} \end{aligned} \tag{15}$$

where

$$\mathbf{K}_k = \mathbf{P}^+_{k+1} \mathbf{H}^T_k \left(\mathbf{H}_k \mathbf{P}^+_{k+1} \mathbf{H}^T_k + \mathbf{J}_k \mathbf{R} \mathbf{J}^T_k\right)^{-1}$$

is the Kalman gain matrix, being $\mathbf{J}_k = J(\boldsymbol{\theta}_k)$ and $\mathbf{R} = diag\left(\mathbf{R}_{\text{gps}}, \mathbf{R}_{\text{usbl}}, R_{\text{depth}}, \mathbf{R}_{\text{prop}}\right)$.

IV. Results

The navigation filter has been tested off-line on real data, collected during the sea-trial of Sept. 12, 2013. In the presented trial, the Typhoon has executed an autonomous surface mission within the La Spezia harbour, consisting in the repetition of a triangle-shaped path with vertices placed in the waypoints WP1, WP2 and WP3. In this area, some battery-operated modems were deployed to build an ad-hoc installation of fixed nodes. Knowing the absolute position of the modems, the vehicle can localize itself using the on-board USBL.

The prediction step of the filter is executed at the rate of the inertial unit, $10Hz$, that is the highest rate among those of the sensors. Since the fixes from the auxiliary sources have different rates, the correction step can not be performed at the same frequency. In particular, the depth and the surge speed information are notified by the respective sensors at the same rate of the inertial unit, so the relative correction is done at each time instant. On the other hand, the USBL measurements have not a periodic time of notification, so the correction of the absolute position is executed as soon as a new measurement becomes available.

A preliminary analysis of the raw data was carried out in order to tune the filter parameters, i.e. the covariance matrixes

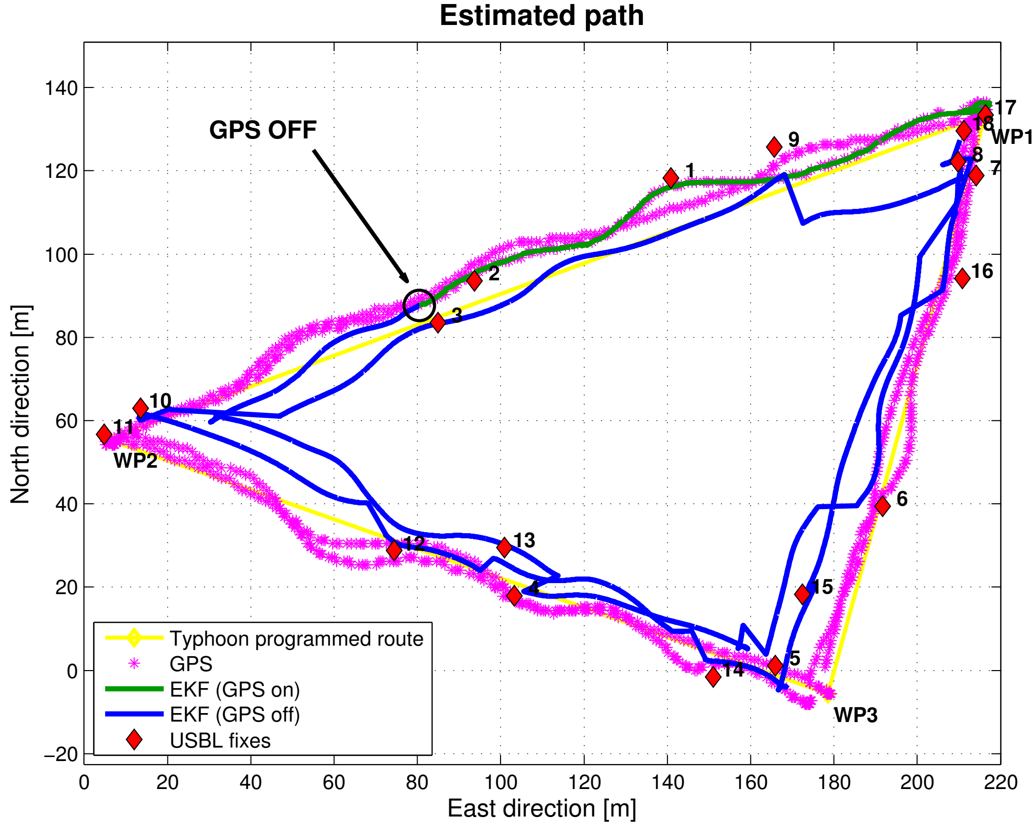

Fig. 2. Estimation of the North-East path. The solid yellow line shows the triangle-shaped path that Typhoon had to follow. In the first part of the path (solid green line), the GPS was used to feed absolute position correction to the estimation filter. During the remaining part (solid blue line), the GPS signal loss is simulated and the forward speed and the USBL measurements only were used as position correction feeds to the estimation filter. The black circle on the map indicates the point where the GPS signal loss begins; starting from here, GPS was used as ground truth only.

TABLE I. ERROR BETWEEN GPS AND USBL FIXES

Fix no.	Error (m)	Fix no.	Error (m)
1	2.27	10	8.48
2	3.38	11	2.15
3	8.18	12	5.94
4	3.04	13	8.44
5	13.23	14	4.26
6	2.91	15	9.83
7	1.89	16	11.55
8	3.11	17	1.32
9	5.09	18	1.16

of the noises. For example, the covariance matrix of the USBL, $\mathbf{R}_{usbl}$, was set by considering the errors between the absolute position evaluated on the basis of the USBL fixes and the corresponding GPS measurement. These errors, reported in our previous work [8], are repeated here in Table I for self-concistency.

Figure 2 shows the North-East path estimated by the designed filter. During the first part of the simulation, represented with a solid green line, the GPS measurements were used as auxiliary corrections in order to calibrate the estimation of the accelerometers bias. Starting from the point indicated by the black circle, the loss of the GPS signal was simulated, as in an underwater mission, and the GPS fixes are used as ground truth only. In this part, the auxiliary sensors used to feed correction to the navigation filter are the USBL, the depth sensor and the surge speed only.

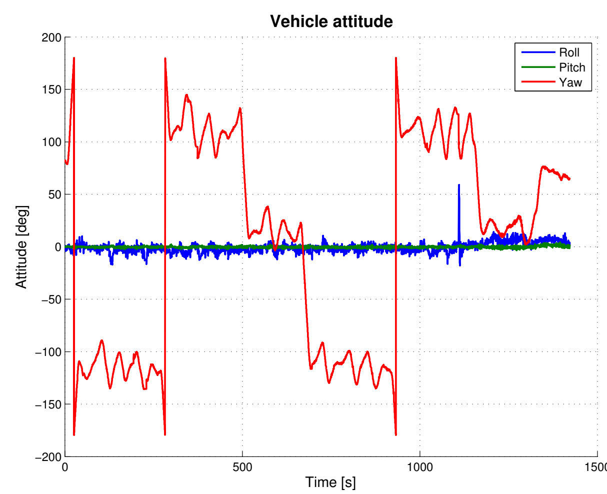

Fig. 3. Attitude estimation obtained by fusing in an AHRS fastion the measurements from the gyros, accelerometers and magnetometers.

Figure 3 illustrates the vehicle attitude parametrized with Euler angles (RPY). The attitude is estimated by the inertial unit on-board the vehicle by integrating the inertial measurements together with the Earth magnetic field measurements into an AHRS fashion.

In Figure 4 is shown the estimate of the accelerometers

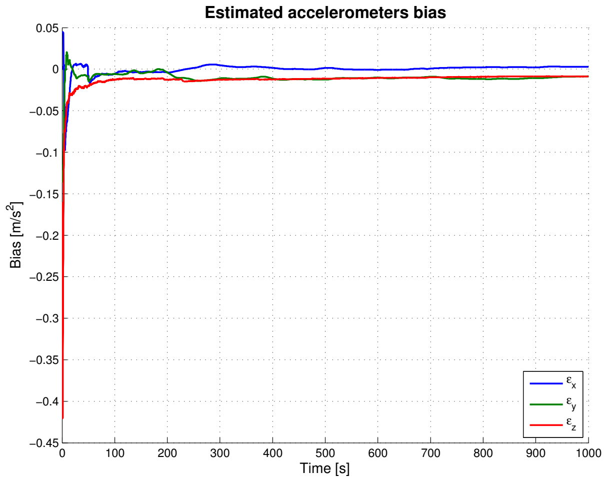

Fig. 4. Kalman Filter estimation of the accelerometers bias.

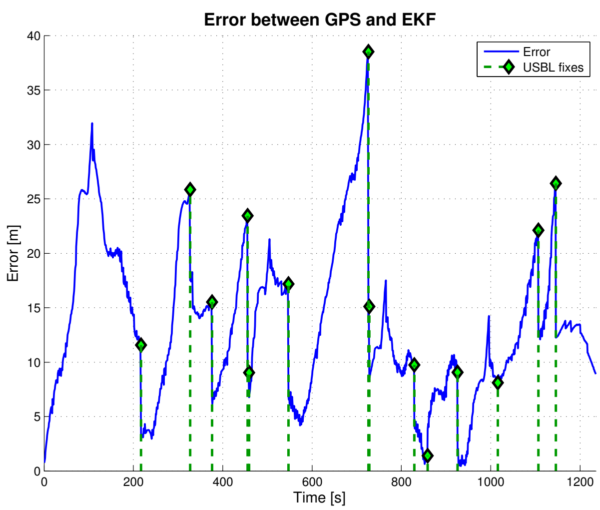

Fig. 5. Norm of the error between the GPS measurement and the estimated position after the simulated GPS signal loss; the green diamonds indicate the time instants of the USBL fixes availability.

bias decomposed in the x-y-z coordinates. We note that, after about 500 seconds, the accelerometers bias converges to its steady-state value.

Figure 5 presents the norm of the error between the estimated path and the GPS position, starting from the begin of the diving simulation. As it can be seen, the acoustic fixes, even if coming at intervals of minutes one from the other, are indeed succesfull in bounding the navigation error and in mitigating, if not zeroing, the error induced by the inertial estimation drift. A time lag in acoustic fixes as in the experiment reported here would indeed be typical in a multi-vehicle situation in which communication and ranging occur in a networked fashion.

V. Conclusion

The paper presented some experimental results of autonomous underwater navigation, based on the fusion of acoustic and inertial measurements. The test was carried out with one low-cost AUV with acoustic communication capabilities, during the CommsNet13 experiment, organized and scientifically coordinated by the NATO S&T Org. Ctr. for Maritime Research and Experimentation. To integrate the navigation data, we first modeled the kinematics of the vehicle and the measurements from the available sensors. Thus, on the basis of the model, an Extended Kalman Filter has been designed to implement the vehicle navigation algorithm. The field results reported shows a performance similar to that of other field experiment with comparable set-up, although in our case we have larger delays from one USBL update to the next. We remark that a low reception rate of the acoustic corrections from the USBL is typical in an acoustic sensors network, and thus in a multi-AUV communication scenario.

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